

## **EXHIBIT 21**

# Econometrics

Legal, Practical, and  
Technical Issues

Second Edition



This volume should be officially cited as:

ABA SECTION OF ANTITRUST LAW,  
ECONOMETRICS, SECOND EDITION (2014)

Cover design by ABA Publishing.

The materials contained herein represent the opinions of the authors and editors and should not be construed to be the views or opinions of the law firms or companies with whom such persons are in partnership with, associated with, or employed by, nor of the American Bar Association or the Section of Antitrust Law unless adopted pursuant to the bylaws of the Association.

Nothing contained in this book is to be considered as the rendering of legal advice for specific cases, and readers are responsible for obtaining such advice from their own legal counsel. This book and any forms and agreements herein are intended for educational and informational purposes only.

© 2014 American Bar Association. All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, electronic, mechanical, photocopying, recording, or otherwise, without the prior written permission of the publisher. For permission contact the ABA Copyrights & Contracts Department, [copyright@americanbar.org](mailto:copyright@americanbar.org) or via fax at (312) 988-6030, or complete the online form at <http://www.americanbar.org/utility/reprint.html>.

Printed in the United States of America.

18 17 16 15 14 5 4 3 2 1

ISBN: 978-1-61438-935-4

Library of Congress Catalog Number: 2014933353

Discounts are available for books ordered in bulk. Special consideration is given to state bars, CLE programs, and other bar-related organizations. Inquire at Book Publishing, ABA Publishing, American Bar Association, 321 N. Clark Street, Chicago, Illinois 60654-7598.

## CONTENTS

<i>Foreword</i> .....	xiii
<i>Preface</i> .....	xv

### Chapter 1

<b>An Introduction to Econometric Analysis</b> .....	1
A. How Econometrics Can Be Helpful in an Antitrust Analysis .....	1
1. Providing Measures of Quantitative Inputs to an Antitrust Analysis .....	2
2. Identifying and Measuring Causal Relationships .....	2
3. Testing Propositions about the Behavior of Companies and Consumers .....	3
B. The Elements of an Econometric Study .....	3
1. Articulating the Question to Be Studied .....	4
2. Considering the Underlying Economics .....	4
3. Collecting Relevant and Useful Data .....	5
4. Formulating and Estimating an Econometric Model .....	5
5. Interpreting the Results .....	5
a. A Word of Caution in Interpreting a “Not Statistically Significant” Result .....	6
b. A Word of Caution in Interpreting a “Statistically Significant” Result .....	7
6. Presenting the Results .....	7
C. Conclusions .....	9

### Chapter 2

<b>Formulating Empirical Questions of Interest</b> .....	11
A. Market Definition and Market Power .....	11
1. Market Definition .....	11
2. Market Power .....	14
B. Merger Analysis .....	15
1. Prospective Analysis .....	15
a. Event Analysis .....	15
b. Demand Modeling .....	16

2. Retrospective Analysis .....	17
C. Damages .....	18
1. Overcharges .....	19
2. Lost Profits .....	20
D. Class Certification .....	21
E. Conclusions .....	22

### Chapter 3

#### The Econometric Framework..... 25

A. Brief History of Statistics and Econometrics: Drawing Inferences about Causal Relationships from Data .....	25
1. Statistics and Randomized Experimental Design .....	26
2. Causality .....	31
3. Econometrics .....	34
a. Structural Models.....	35
b. Econometric Estimation Methods for Structural Models .....	36
c. Reduced Form Models .....	38
d. Analysis of Natural Experiments .....	39
e. Specification Testing .....	41
B. Conclusions .....	43

### Chapter 4

#### Collecting Relevant and Useful Data..... 45

A. Introduction .....	45
B. Nature of Economic Data .....	45
1. Cross-Sectional Data.....	46
2. Time Series Data.....	46
3. Data with Both Cross-Sectional and Time Series Data .....	47
C. Types of Data.....	48
1. Accounting Data .....	48
2. Transaction Data .....	49
3. Scanner Data .....	51
4. Survey Data .....	54
D. Sources of Data.....	55
1. Ordinary Course Data Maintained by the Parties .....	55
2. Commercial Databases .....	56

3. Publicly Available Data .....	57
E. Data Considerations That May Affect the Analysis .....	57
1. Outliers .....	58
2. Missing Observations .....	59
3. Small Sample Size .....	60
4. Omitted Variables .....	61
5. Data Aggregation.....	61
6. Admissibility of Data Analysis.....	63
F. Conclusions .....	64

### Chapter 5

#### Specifying and Estimating an Appropriate Econometric

#### Model ..... 65

A. Basics of an Econometric Model .....	65
B. Key Considerations in Specifying an Econometric Model.....	68
1. Examples of How Available Data Affects the Specification of Econometric Models.....	68
a. Fixed Effects .....	68
b. Benchmark Models .....	70
(1) Cross-Sectional Data and Contemporaneous Benchmark Models.....	70
(2) Time Series Data and Before-and-After Models.....	71
(3) Panel Data and Difference-in-Differences Models .....	72
2. Multiple Equation Models .....	75
3. Analyzing Consumer Preferences Using Discrete Choice Models .....	77
C. Some Potential Modeling and Estimation Issues.....	79
1. Functional Form.....	80
2. Omitted Variables .....	81
3. Multicollinearity .....	82
4. Endogeneity .....	83
5. Homoskedasticity, No Autocorrelation, and Other Assumptions of the Classical Linear Regression Model .....	86
D. Conclusions .....	88

### Chapter 6

#### Interpreting Regression Results ..... 89



A.	Understanding Regression Results .....	89
1.	Evaluating the Specification of the Regression Model .....	89
2.	Interpreting Regression Coefficients in an OLS Regression Model .....	90
3.	Interpreting Regression Coefficients in a Discrete Choice Model .....	92
4.	An Example: Estimating Price Overcharges .....	94
B.	Hypothesis Testing .....	95
1.	Statistical Tests .....	96
a.	Hypotheses about a Single Parameter: t-tests .....	96
b.	Testing Multiple Hypotheses: F-tests .....	97
2.	Statistical Significance .....	98
3.	Practical Significance .....	99
C.	Specification Testing .....	99
1.	Goodness of Fit .....	100
a.	R-Squared and Adjusted R-Squared .....	100
b.	Tests to Compare Two Model Specifications .....	101
2.	Modeling Issues That Could Affect the Reliability of the Results .....	102
a.	Omitted Variables .....	102
b.	Multicollinearity .....	104
c.	Endogeneity .....	106
d.	Heteroskedasticity and Autocorrelation .....	107
e.	Special Considerations in Time Series Analysis .....	109
D.	Data Issues That Could Affect the Reliability of the Results .....	111
1.	Insufficient Data .....	111
2.	Outliers .....	112
3.	Missing Data .....	113
4.	Measurement Errors .....	114
5.	Improper Aggregation of the Data .....	115
E.	Conclusions .....	116

## Chapter 7

### Presentation of Econometric Analyses..... 117

A.	Knowing Your Audience .....	117
1.	The Client .....	118
2.	Opposing Counsel and Experts .....	118

3.	The Judge .....	119
4.	The Jury .....	119
B.	Presenting Econometric Evidence in an Expert Report .....	120
1.	Preparing an Econometric Primer .....	120
a.	Laying Out Basic Concepts of Econometrics .....	120
b.	Introducing More Advanced Concepts of Econometrics .....	123
2.	Describing the Underlying Data .....	124
3.	Describing the Choice of Regression Models .....	125
4.	Discussing the Variables .....	127
5.	Presenting the Results of an Econometric Analysis .....	129
6.	Presenting the Robustness of Econometric Results .....	132
7.	Presenting Critiques of the Econometric Analysis .....	134
8.	Demonstrative Exhibits .....	137
C.	Presenting Econometric Evidence at Trial .....	138
1.	Trial Testimony .....	139
2.	Approaches and Examples .....	142
D.	Conclusions .....	146

## Chapter 8

### Evidentiary Issues..... 147

A.	Admissibility Under <i>Daubert</i> .....	147
1.	Introduction .....	147
2.	Qualifications of the Expert .....	149
3.	Relevance .....	151
4.	Sufficient Facts or Data .....	153
a.	Weaknesses in the Underlying Data .....	153
b.	The "Fit" Between the Data and the Analysis .....	158
5.	Reliability .....	160
a.	Analysis Can Be Empirically Tested .....	161
b.	Analysis Has Been Published or Subjected to Peer Review .....	163
c.	Analysis Presents an Acceptable Known or Potential Rate of Error .....	164
d.	Analysis Is Generally Accepted in Scientific Community .....	165

e. Analysis Grows Naturally and Directly Out of Research Conducted Independently of the Litigation.....	169
f. Analysis Includes All Relevant Variables .....	170
g. Analysis Treats Outliers Appropriately .....	176
B. Burden of Proof .....	176
C. Additional Issues Regarding the Admissibility of Econometric Evidence .....	180
1. Appellate Standard of Review .....	181
2. Preserving the Issue of Admissibility on Appeal.....	181
3. Using <i>Daubert</i> Factors after Admission .....	183
4. Using <i>Daubert</i> Factors at Class Certification Stage .....	183
D. Conclusions .....	186

## Chapter 9

<b>Expert Discovery.....</b>	<b>187</b>
A. Discovery of Testifying Experts .....	187
1. Document and Data Discovery .....	188
2. Deposition of a Testifying Expert.....	194
a. Expert's Qualifications as an Econometrician.....	195
b. Potential Data Problems.....	196
c. Violations of Statistical Assumptions.....	197
d. Prior Inconsistent Econometric Results .....	197
e. Discussions with Other Experts .....	198
B. Discovery of Non-Testifying Experts.....	198
C. Other Discovery Issues .....	205
1. Discovery from Non-Parties .....	205
2. Discovery from the Government .....	207
a. Governmental Privileges .....	207
b. The Freedom of Information Act .....	211
3. Consequences of Non-Compliance.....	212
D. Conclusions .....	214

## Chapter 10

<b>Applying Econometrics to Assess Market Definition and Market Power.....</b>	<b>215</b>
A. Legal and Economic Framework.....	215
B. Market Definition .....	217

1. Estimating the Elasticity of Demand .....	218
a. Modeling Different Types of Consumer Preferences .....	220
(1) Product Space Models .....	222
(2) Characteristic Space Models .....	224
b. Applying the Elasticity of Demand to the Hypothetical Monopolist Test.....	227
c. Applying the Elasticity of Demand to the Critical Loss Test .....	229
d. Elasticities of Marshallian Demand and Residual Demand .....	229
2. Assessing the Relationship between Prices of Competing Products or Prices in Different Regions.....	232
a. Price Correlations and Other Non-Causal Approaches.....	233
(1) Price Correlations .....	233
(2) Price Equality Tests.....	234
b. More Advanced Econometric Approaches .....	235
(1) Cointegration Analyses .....	235
(2) Speed of Adjustment Tests.....	236
(3) Price Responsiveness Tests .....	237
(4) Natural Experiments.....	238
3. Analysis of Shipment Data .....	240
C. Market Power .....	241
1. Market Share, Concentration, and Entry.....	241
a. Market Share as a Measure of Market Power .....	241
b. Constructing Market Share from Market Data .....	242
c. Calculating the Herfindahl-Hirschman Index .....	243
d. Econometric Models of Market Structure and Entry .....	245
2. Supracompetitive Profit Margins.....	247
a. Profit Margins as a Measure of Market Power .....	247
b. Measuring Profit Margins Using Financial and Accounting Data .....	248
c. Estimating Profit Margins from the Inverse Demand Elasticity .....	249
3. Supracompetitive Pricing.....	252
a. Supracompetitive Pricing as a Measure of Market Power .....	252
b. Identifying Benchmark Prices.....	253

c. Estimating Supracompetitive Pricing Using a Benchmark Price.....	253
d. Estimating Supracompetitive Pricing Using Reduced-Form, Price-Concentration Models.....	257
e. Estimating Supracompetitive Pricing Using a Structural Regression Model.....	261
D. Conclusions .....	262

## Chapter 11

### Applying Econometrics to Merger Review..... 265

A. Legal and Regulatory Framework .....	265
B. Empirical Methods for Merger Analysis .....	267
1. Diagnostic Techniques Used to Identify Potentially Problematic Mergers.....	267
2. Natural Experiments .....	268
3. Merger Simulation .....	271
a. Overview.....	271
b. Implementation .....	273
c. Reliability of Merger Simulation.....	276
C. Estimating the Price Effects of Proposed Transactions .....	277
1. Analyses of Potential Unilateral Effects .....	277
a. Whole Foods and Wild Oats .....	277
b. Hospital Mergers.....	281
2. Analyses of Potential Coordinated Effects .....	284
a. The Cruise Lines Mergers.....	284
b. The Australian Mortgage Market.....	288
c. HP-Compaq .....	291
D. Estimating the Competitive Effects of Consummated Transactions.....	295
E. Conclusions .....	299

## Chapter 12

### Applying Econometrics to Estimate Damages..... 301

A. Non-Econometric Approaches to Damages.....	302
B. The Econometric Approach to Damages .....	305
C. Overview of Legal and Economic Framework.....	306
D. Estimating Damages Using Econometric Techniques.....	308

1. Before-During Approach .....	312
a. “Dummy Variable” Models Versus “Prediction” Models .....	312
2. Before-During-After Models .....	316
3. Benchmark and Difference-in-Differences Approaches to Damages .....	319
E. Key Considerations in Creating and Evaluating Econometric Models of Damages.....	320
F. Case Study: Applying the Before-During and Benchmark Approaches .....	325
1. Background.....	325
2. Economic Model, Data, and Choice of Explanatory Variables.....	326
3. Assumption of Liability and Causation .....	329
4. Stationarity of the Time Series .....	329
5. The Before-During Approach with the Dummy Variable Model.....	331
6. The Before-During Approach with the Prediction Model .....	336
7. Benchmark Approach .....	338
G. Conclusions .....	340

## Chapter 13

### Applying Econometrics to Address Class Certification..... 341

A. Legal and Economic Framework.....	342
1. Demonstrating Common Proof of Injury .....	343
2. Proving Damages.....	348
B. Econometric Analysis in Class Certification Disputes.....	349
1. Regression Analysis.....	351
a. Assessing the Extent of Customer-Specific Pricing .....	351
b. Testing for the Existence of a Common Price Structure.....	354
c. Testing for Classwide Impact .....	355
(1) Regression Analysis to Measure Average Impact.....	356
(2) Regression Analysis as a Test of Classwide Impact Using a Common Method of Proof .....	357
d. Summary .....	360
C. Case Studies.....	361



1. Identifying Customer-Specific Pricing: <i>In re Graphics Processing Units Antitrust Litigation (GPU)</i> .....	361
a. Case Background .....	361
b. Plaintiffs' Expert's Econometric Analysis .....	362
c. Defendants' Expert's Econometric Analysis .....	365
d. Case Study Insights .....	366
2. Case Study of Testing for Classwide Impact: Plastic Additives .....	367
a. Case Background .....	367
b. Plaintiffs' Expert's Econometric Analysis .....	368
c. Defendants' Expert's Econometric Analysis .....	368
d. Case Study Insights .....	369
D. Conclusions .....	370
<b>Appendix and Glossary .....</b>	<b>371</b>
A. Introduction .....	371
B. Correlation Between Two Variables .....	371
C. Classic Linear Regression Analysis .....	373
1. Ordinary Least Squares Regression .....	374
2. Simple Regression Models with One Explanatory Variable .....	376
3. Regression Models with Multiple Independent Variables .....	377
4. The Regression Residual .....	380
D. The Precision of Regression Results .....	380
1. Standard Errors .....	380
2. Confidence Intervals .....	383
3. t-Statistics, Statistical Significance and Hypothesis Testing .....	384
4. Testing the Difference Between Two Means .....	387
5. Goodness-of-Fit .....	388
E. Glossary .....	390
Table of Cases .....	423
Index .....	435

## FOREWORD

The Section of Antitrust Law is pleased to publish *Econometrics: Legal, Practical, and Technical Issues* (Second Edition). Econometrics plays a central role in modern antitrust litigation and merger analysis, and economic experts are regularly the star witnesses in court and before the enforcement agencies. With the advent of increasingly sophisticated econometric techniques, it is more important than ever for practicing antitrust lawyers to have a complete and current understanding of these techniques and how to use them. This volume explains in plain language the technical elements of econometric analyses and the situations in which they can be properly deployed. Because nine years have passed since the first edition, this book is much more than just an update. It has been substantially rewritten, reorganized and expanded to reflect the increasingly important and complex roles that econometrics and economics play in antitrust analysis and litigation.

The Antitrust Section is grateful to the many economists and lawyers who worked on this book. The Section is especially grateful for the exceptional and tireless work of the book's editor, Lawrence Wu, who organized and supervised a talented cadre of economists (with the help of some lawyers) to produce a book that is both technically sound and accessible to non-economists. He was assisted by Charles Biggio, who is the co-editor of this volume. The Section also thanks the individuals who devoted so much time to researching, drafting and revising the substantial new content in this book. They, as well as many others who made significant contributions, are credited by name in the Preface.

February 2014

Christopher B. Hockett  
Chair, Section of Antitrust Law  
American Bar Association  
2013-2014



from the effect of the other on the dependent variable. The model cannot be estimated.

Multicollinearity occurs when two or more explanatory variables are highly but not perfectly correlated. A model exhibiting multicollinearity can be estimated, unlike a perfectly collinear model, but OLS coefficient estimates may be difficult to interpret. First, the OLS-estimated coefficient of one variable, say  $\beta_1$ , is interpreted to measure the change in  $Y$  when  $X_1$  changes, assuming all the other independent variables are held constant. However, if two variables  $X_1$  and  $X_2$  are highly correlated,  $X_2$  is likely to change predictably whenever  $X_1$  changes. Second, as the correlation between two or more explanatory variables increases, so do the standard errors of OLS-estimated regression coefficients. Large standard errors reduce the precision of OLS coefficient estimates and make it more difficult for an expert to reject the null hypothesis of a hypothesis test. Under these circumstances, failure to find a relationship between an explanatory variable and the dependent variable does not necessarily mean that none exists. It may be impossible for an expert to determine whether a relationship exists given the data available.

To determine if multicollinearity exists, an expert may look at the standard errors of OLS coefficient estimates. If they are high, the expert can re-estimate the model excluding one or more of the explanatory variables suspected of causing multicollinearity. Lower standard errors on the remaining variables are suggestive of multicollinearity in the original model. The dropped variable(s) may cause the re-estimated model to suffer from omitted-variable bias.

**Normal Distribution:** A probability distribution that OLS parameter estimates are assumed to follow. If a variable  $X$  is normally distributed, its probability distribution exhibits a bell-shaped pattern as shown in Figure 5.

**Null Hypothesis:** The assumption tested in a hypothesis test, denoted as  $H_0$ .

**Omitted-Variable Bias:** An omitted variable is a relevant explanatory variable that is excluded from a regression model, leading to a misspecification of the relationship between the dependent variable  $Y$  and the included independent variables  $X$ . When data on an

omitted variable are available, the problem is readily corrected by adding the omitted variable to the model.

When data on the omitted variable are not available, the problem is more serious. The omitted variable is then automatically included in the error term of the model. If one or more included explanatory variables are correlated with the omitted variable, as is often the case, those variables are correlated with the error term. The included variables may be credited with an effect actually attributable to the omitted variable; this over- or underestimation of OLS regression coefficients is known as omitted-variable bias. The estimated coefficients are both biased and inconsistent (in large samples).

As in the car sales-GDP-gas price model presented above, suppose annual U.S. car sales depend on both GDP and the nationwide average retail gas price:

$$Y_i = \alpha + \beta_1 X_{1i} + \beta_2 X_{2i} + \varepsilon_i,$$

where  $Y$  is car sales,  $X_1$  is GDP and  $X_2$  is gas price. If gas price data are not available,  $X_2$  is omitted from the regression and the model becomes

$$Y_i = \alpha + \beta_1^* X_{1i} + \varepsilon_i^*.$$

To the extent that GDP and gas price are correlated, the coefficient on GDP in the latter equation,  $\beta_1^*$ , picks up some of the effect of the omitted gas price variable on car sales and thus is biased. Generally the only case in which omitted-variable bias disappears is when an omitted variable is uncorrelated with *all* included explanatory variables, an unlikely scenario.

There are several ways to address omitted-variable bias when data on the omitted variable(s) are not available. One possible method is to obtain data on a proxy variable related to the omitted variable, if such a proxy variable exists. For example, IQ is often used as a proxy for unobservable “ability” in models estimating wages as a function of education, experience and ability. If the omitted variable can be assumed not to change over time, fixed effects or differencing techniques can be used to estimate the model (see, e.g., Wooldridge pp. 438-450). Instrumental variables and two-stage least squares estimation techniques can also resolve omitted variables problems and yield consistent parameter estimates. If none of these corrections are applicable, an expert should at the very least be prepared to discuss potential omitted variables and the likely direction and size of any omitted-variable bias on the variables included in the model.

It may be possible for an expert to account for omitted-variable bias qualitatively if the expert has knowledge about the relationship between the omitted variable and the explanatory variable(s) of interest. For example, suppose in a sex discrimination pay case that women are on average more skilled than men. However, the expert cannot obtain quantifiable data reflecting necessary job skills. The expert's regression of the wage rate on years of experience and a gender indicator variable suggests that men are paid more than women with the same experience. Since differences in skill level have not been accounted for, the expert may reasonably conclude that the estimated wage difference is a conservative estimate of the true difference.<sup>44</sup>

Both the direction and the size of omitted-variable bias are important. When there are multiple explanatory variables included in a model, it is not always straightforward to determine the direction of omitted-variable bias on a particular included variable. Even if the omitted variable(s) are only correlated with one included explanatory variable, generally the coefficients on all included explanatory variables are biased.<sup>45</sup>

**Ordinary Least Squares (OLS) Regression:** OLS is most basic method of regression analysis and defines a linear relationship between a dependent and one or more independent variables. An OLS model specifying a dependent variable as a linear function of k independent variables is written as  $Y_i = \alpha + \beta_1 X_{1i} + \beta_2 X_{2i} + \dots + \beta_k X_{ki} + \varepsilon_i$ .

**Outlier:** An outlier is a data point that lies far from the regression line that fits all other points in a data set. OLS regression can be sensitive to extreme data points. Consider Figures 6 and 7:

44. Daniel L. Rubinfeld, *Reference Guide on Multiple Regression*, in FEDERAL JUDICIAL CENTER, REFERENCE MANUAL ON SCIENTIFIC EVIDENCE 189 (2d ed. 2000).  
45. For a derivation of the size of omitted-variable bias, see PINDYCK & RUBINFELD, *supra* note 27, at 184-186.

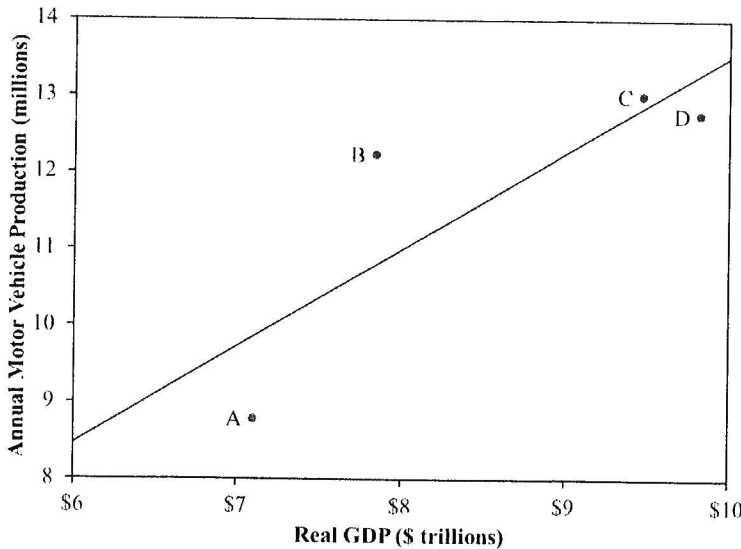


Figure 6. U.S. Motor Vehicle Production and GDP

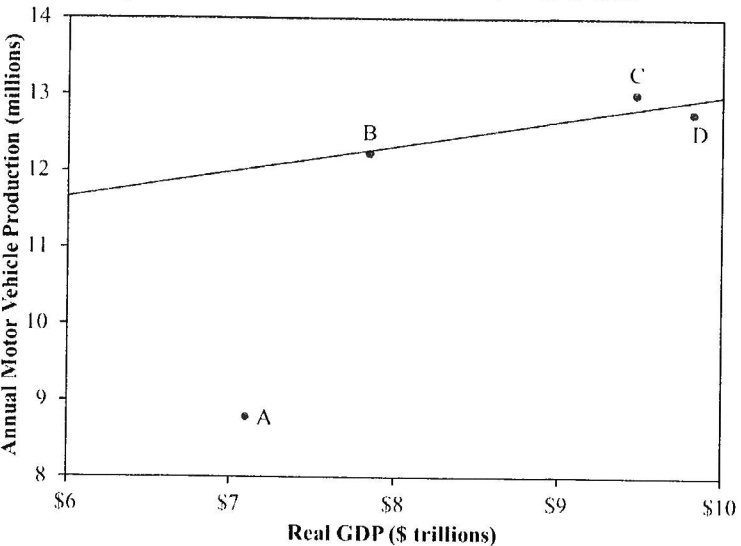


Figure 7. U.S. Motor Vehicle Production and GDP

The OLS regression line in Figure 6 describes the best-fitting relationship between Points A, B, C and D. Figure 7 shows the best-fitting OLS regression line describing the relationship between Points B, C and D only. The flatter regression line in Figure 7